

ORION EXPLORATION MISSION ENTRY INTERFACE TARGET LINE

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The Orion Multi-Purpose Crew Vehicle is required to return to the continental United States at any time during the month. In addition, it is required to provide a survivable entry from a wide range of trans-lunar abort trajectories. The Entry Interface (EI) state must be targeted to ensure that all requirements are met for all possible return scenarios, even in the event of no communication with the Mission Control Center to provide an updated EI target. The challenge then is to functionalize an EI state constraint manifold that can be used in the on-board targeting algorithm, as well as the ground-based trajectory optimization programs. This paper presents the techniques used to define the EI constraint manifold and to functionalize it as a set of polynomials in several dimensions.

INTRODUCTION

The Orion Multi-Purpose Crew Vehicle (MPCV) is designed to carry crew into trans-lunar space and return them safely to Earth. For a low lift-to-drag ratio capsule such as Orion, the Entry Interface (EI) position and velocity state is the main driver of the entry performance. EI is defined at a geodetic altitude of 400 kft. The entry corridor defines the allowable set of the 5 other position and velocity states: 1) geodetic latitude, 2) longitude, 3) azimuth, 4) flight path angle, and 5) speed. This entry corridor is bounded by several constraint manifolds.

For nominal return scenarios, a landing zone off the coast of San Diego must be achieved anytime during the month. Due to the Earth-Moon geometry, the geodetic latitude of the EI state is driven by the time of departure from the Moon. The orbit of the Moon about the Earth then defines a range of possible EI geodetic latitudes.¹ This drives a maximum skip entry ranging capability of up to 4800 nmi. The longitude of the EI state can be controlled through the time-of-flight of the return trajectory back from the Moon. To accommodate weather-driven retargeting, Orion must be capable of diverting short of a 1200 nmi radius storm system centered at the landing zone. Due to sensed acceleration limits on the crew, the minimum ranging capability of Orion is limited to 1300 nmi. This limits the minimum nominal entry range to 2500 nmi in order to preserve the weather divert capability.

For a given EI geodetic latitude/longitude pair, there is a range of allowable azimuth values pointing toward the landing site and within the crossrange capability of the vehicle. In addition, the azimuth is bounded to a minimum value of 0° to avoid retrograde entries, as these would be excessively stressing on the Thermal Protection System (TPS). By proper selection of the crossrange at EI, it is possible to control both the direction and timing of the first bank reversal.

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The maximum value of the EI flight path angle (i.e. shallowest flight path angle) is constrained to preserve an untargeted ballistic entry downmode option. In this downmode, the vehicle is spun at a constant bank rate about the velocity vector in order to null the lift vector. The statistical spread of landing locations is called the footprint. The footprint of such an entry is a strong function of the maximum altitude rate encountered during flight. When the altitude rate goes positive, trajectory lofting occurs and the footprint grows in size. If the altitude rate gets too large, skip out of the atmosphere can occur. To aid in recovery operations of such a scenario, it is desired to limit the landed footprint. This is achieved by setting a maximum altitude rate of 0 ft/s for ballistic downmode entries. The mass of the Orion heat shield is driven by steeper flight path angles. To limit TPS mass, the minimum flight path angle (i.e. steepest flight path angle) is constrained to be 0.15° steeper than the ballistic lofting limit.

The EI speed is bounded by the set of all possible Earth return scenarios. Orion is required to provide survivable entry from a wide range of trans-lunar and abort-to-orbit trajectories. Thus, the entry corridor must be defined for entry speeds from Low Earth Orbit (LEO) all the way up to lunar returns. This range is roughly 26247 to 36089 ft/s (8 to 11 km/s).

Finally, for a given EI state to be valid, it must provide for safe Service Module (SM) disposal. The SM debris must be placed in the ocean at least 25 nmi from any United States land mass, and 200 nmi from any land mass of another country.

The EI state must be targeted to ensure that all requirements are met for all possible return scenarios, even in the event of no communication with the ground to provide an updated EI target. The challenge then is to functionalize an EI state constraint manifold that can be used in the on-board targeting algorithm, as well as the ground-based trajectory optimization programs. Past approaches have modeled portions of the constraint manifolds as 6-degree order polynomial target lines.^{2,3} This paper builds upon this idea and expands it to include all relevant entry constraints. The techniques used to define the EI constraint manifold and to functionalize it as a set of polynomials in several dimensions is discussed in the following sections.

EI TARGET LINE CRITERIA

To support Exploration Mission 1 and 2 (EM1/2) mission planning and on-board targeting, it is desired to define a target line (technically a target surface) through the viable 5-dimensional EI target space. In addition to the constraints discussed above, the target line should also meet the following criteria:

- 1. Minimize the Range-to-Target**

It is desired to fly 2500 nmi entries whenever possible.

- 2. Choose Flight Path Angle to Limit Ballistic Lofting**

The shallow side of the EI flight path angle corridor is constrained to limit the chances of a positive altitude rate during a ballistic entry. This reduces the footprint size of a ballistic entry and makes it easier to locate the crew after an emergency entry.

- 3. Continuously Differentiable Target Line**

It is desired that the target line be smooth with no discontinuities. This is desired for both numerical optimizers used for mission planning and for the targeting algorithm used on-board Orion.

4. Functionalized Target Line

It is desired that the target line be functionalized in the form of a polynomial or other function in order to avoid table look-ups.

The entry corridor constraints can be separated into two types. The first type defines the Earth-Centered, Earth-Fixed (ECEF) position vector and the orientation of the entry velocity about that vector. These are called horizontal constraints and define a relationship between the geodetic latitude, longitude, and azimuth. The second type defines the component of the velocity vector along the ECEF position vector. These are called vertical constraints and define a relationship between the flight path angle, speed, geodetic latitude, and longitude. The horizontal and vertical constraints can be treated independently of each other.

Horizontal Target Line

The Lunar Return Entry Interface Tool was developed to define all acceptable combinations of geodetic latitude, longitude, and azimuth angles that result in an acceptable entry to a given landing site. This includes considerations of the desired initial downrange and crossrange to the target, constraints against retrograde entries, and protection for proper Service Module (SM) disposal. Reference 4 has more details of how these points are defined.

Figure 1 shows the allowable horizontal EI zone and horizontal EI target line for a landing site off the coast of San Diego. The green shaded region shows acceptable geodetic latitude and longitude combinations. The green arrows indicate the acceptable azimuth for a given pair of geodetic latitude and longitude. The allowable EI zone is defined for ranges between 2500 and 4800 nmi from the landing site. The irregular shape and cutouts are due to SM disposal constraints around Pacific islands.

The horizontal EI target line is shown as a blue line with blue arrows indicating the azimuth along the line. The target line spans the entire allowable geodetic latitude space while also minimizing the range as much as possible by following the 2500 nmi arc before blending into the southern latitude region. The blending is defined so that the partial derivative of longitude with respect to the geodetic latitude does not become excessively large. This is done to avoid numerical issues when functionalizing the line and to ensure that the target line is continuously differentiable. Note that between 14° and 21.9° latitude, the line is invalid as it does not meet SM disposal requirements near the Hawaiian Islands. In this region, slight adjustments from the target line would be needed in practice. This is an acceptable trade in order to define a smooth curve.

Vertical Target Line

The vertical target line defines the EI flight path angle target as the point where the maximum altitude rate for a ballistic entry is very near zero (ballistic lofting). The value of this flight path angle is dependent on the geodetic latitude at EI (due to the oblateness of the Earth), the azimuth at EI (due to the speed relative to the rotating atmosphere), the longitude (due to atmospheric variations across the globe), and the speed at EI. To allow the vertical target line to be independent of the horizontal target line and applicable to both targeted and untargeted abort entries, it is required to define this constraint manifold for a wide range of EI states.

Table 1 lists the set of grid points used to define this multi-dimensional constraint manifold. At each unique combination of speed/latitude/longitude/azimuth, a numerical root solving iteration was

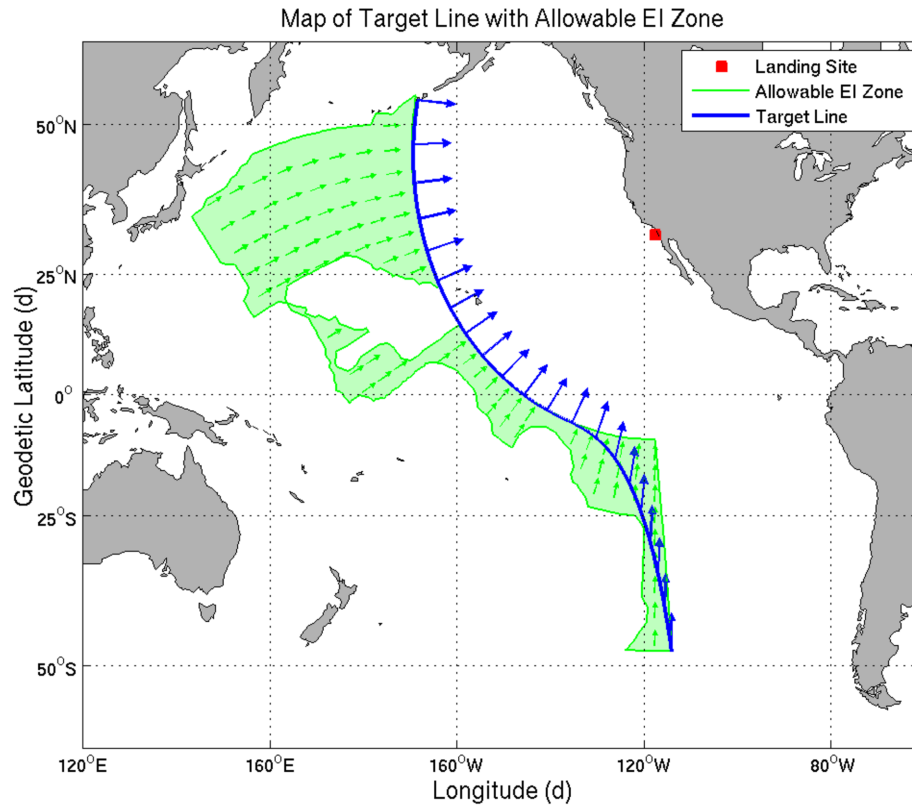


Figure 1. Map of Horizontal Target Line and Allowable EI Zone

performed to find the flight path angle value where ballistic lofting first occurs. Figure 2 shows a map of the unique sample points. The blue arrows show the azimuth sweeps at each unique geodetic latitude and longitude pair.

Table 1. Independent Variables for Ballistic Lofting Definition

Variable	Min Value	Max Value	Increment	# of Points
Inertial Velocity Magnitude	25591 ft/s (7.8 km/s)	37730 ft/s (11.5 km/s)	<i>In general:</i> 328 ft/s (100 m/s) <i>Near 36089 ft/s (11 km/s):</i> 164 ft/s (50 m/s)	37
Geodetic Latitude	-60°	60°	15°	9
Longitude	-150°	180°	30°	12
Inertial Topocentric Azimuth	-165°	180°	15°	24
Total Unique Combinations				95904

The flight path angle limit has a weak dependence on longitude. In order to remove this dependence, for each unique set of geodetic latitude/azimuth/speed, the minimum flight path angle across

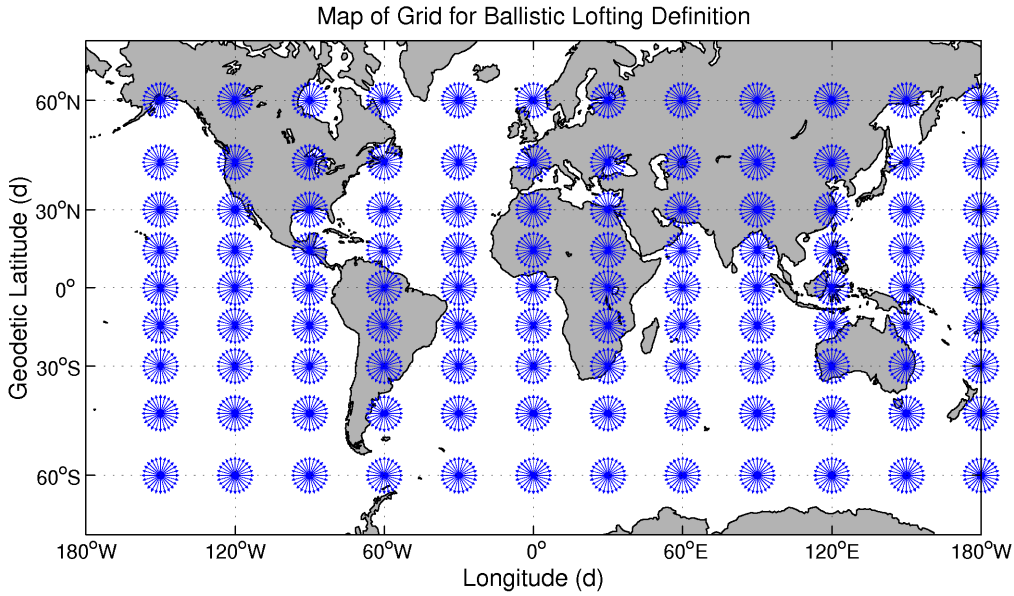


Figure 2. Map of Grid for Ballistic Lofting Definition

all longitude values is taken. This can be visualized as moving all the "columns" of azimuth sweeps in Figure 2 to be on top of each other, with the minimum value of flight path angle retained for each latitude/azimuth. The minimum value is used to ensure that the ballistic lofting limit is never exceeded for any longitude.

Figure 3 shows the resulting flight path angle surface. Figure 4 shows the error introduced by this down sampling. Each azimuth sweep is color coded to show the difference between the actual flight path angle value and the minimum flight path angle surface. The color bar shows the difference above the minimum flight path angle value. It can be seen that the difference from the minimum flight path angle surface is very small for most of the globe. It is nearly zero in the southern hemisphere and is largest over the landmass of Russia where the ballistic lofting limit is almost 0.06° larger than the minimum value. This is likely due to variation in the atmospheric model, which is a function of both the altitude and the position over the Earth. It is unlikely that Orion would enter over Eastern Asia, and if it did the minimum flight path angle surface would still ensure a ballistic entry capability. For this reason, the minimum flight path angle surface is deemed an acceptable approximation of the ballistic lofting limit in order to remove the dependence on longitude.

Now consider the change in flight path angle limit with respect to the entry speed shown in Figure 5. The left subplot shows the inertial topocentric flight path angle as a function of the inertial velocity magnitude. For each velocity value, the flight path angles for all the latitude/longitude/azimuth points are plotted. This shows the spread in flight path angle for each velocity. The subplot on the right shows the difference between the flight path angle for each latitude/longitude/azimuth/speed point relative to the flight path angle value for each latitude/longitude/azimuth point at an inertial velocity magnitude of 36089 ft/s (11 km/s). Let the flight path angle values at 36089 ft/s (11 km/s) be called the basis values. It can be seen that the flight path angle differences relative to the basis collapses to small values at large velocities and increases as the velocity decreases.

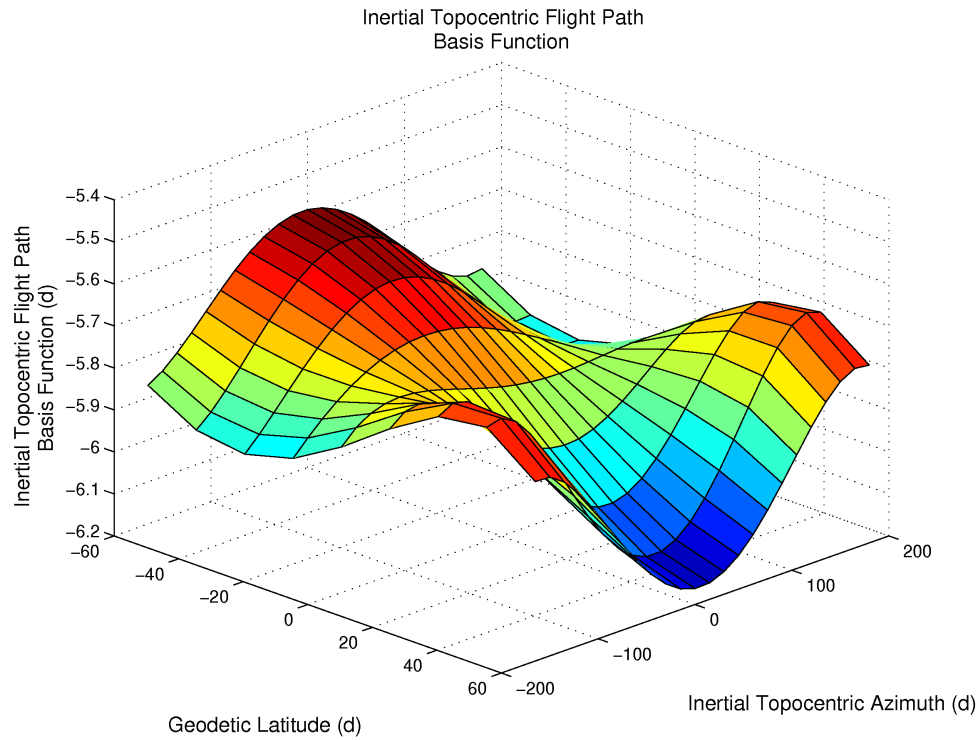


Figure 3. Inertial Topocentric Flight Path: Basis Function

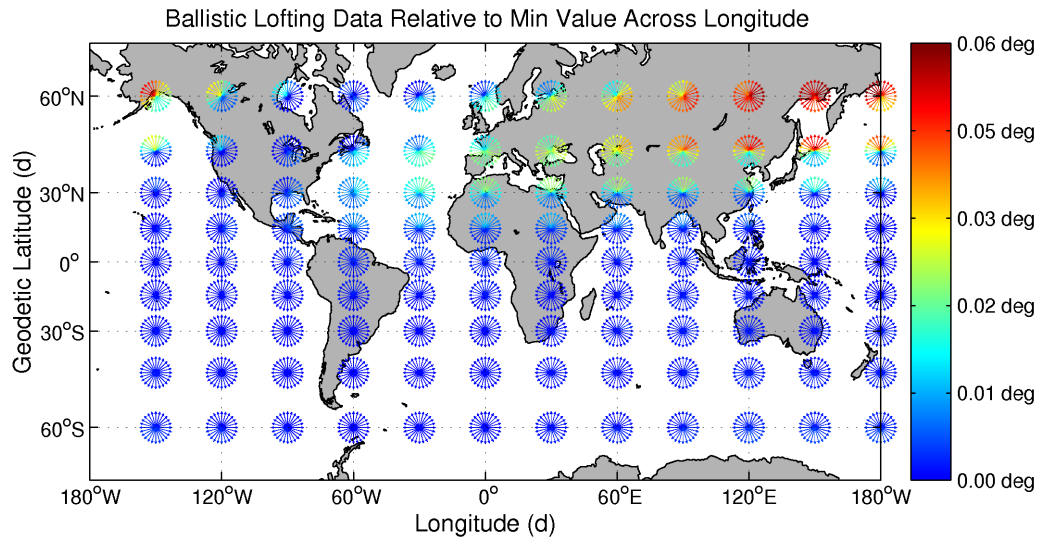


Figure 4. Ballistic Lofting Data Relative to Min Value Across Longitude

Define the delta function as the average value of this difference at each velocity point. The delta function is shown as a red line in the right plot. Note that the delta function is an approximation of the change in the flight path angle with respect to entry speed. The approximation is very good

at high velocities and decreases in accuracy at lower velocities. At high velocities near lunar entry speeds, the flight path angle corridor of Orion has very small tolerances due to heat shield design limits. As the entry speed decreases, the heating environment decreases and the tolerances on flight path angle open up. Thus, the delta function defined here is a reasonable simplification of the dependence of flight path angle on entry speed.

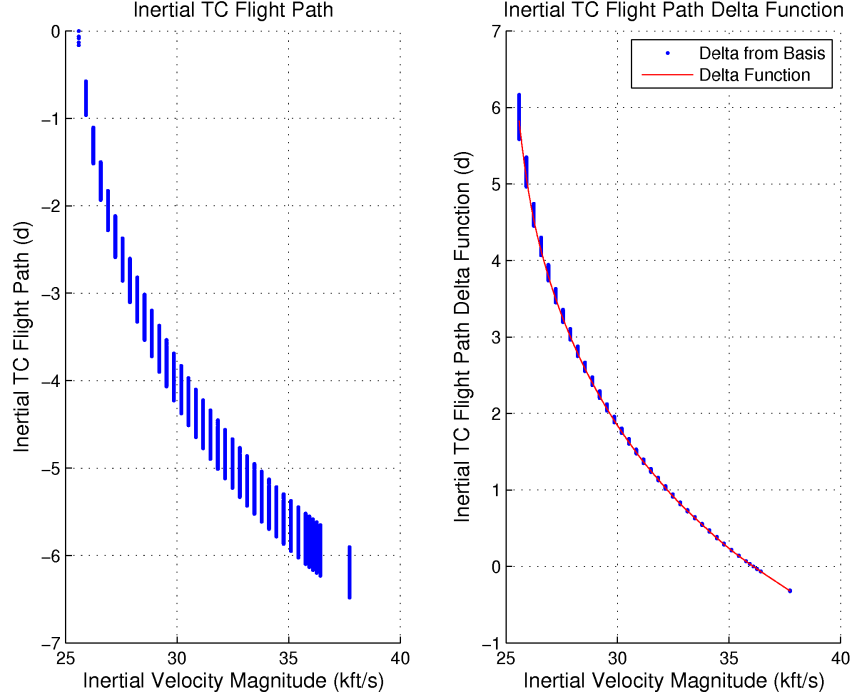


Figure 5. Definition of Flight Path Angle Delta Function

The vertical target line can now be defined in two parts: a basis surface function (β_γ) which is dependent on geodetic latitude and inertial topocentric azimuth, and a delta function (Δ_γ) which is dependent on inertial velocity magnitude. The basis function is defined as the minimum flight path angle surface shown in Figure 3 and gives the desired flight path angle for 36089 ft/s (11 km/s) inertial velocity magnitude anywhere on the Earth. The delta function gives the variation in the desired flight path angle as a function of inertial velocity magnitude. The two components are approximations which capture the variation in ballistic lofting flight path angle over the oblate and rotating Earth. The flight path angle target can then be computed from equation 1:

$$\gamma = \beta_\gamma + \Delta_\gamma \quad (1)$$

where

- γ = Inertial topocentric flight path angle
- β_γ = Basis function of inertial topocentric flight path angle
- Δ_γ = Delta function of inertial topocentric flight path angle

EI TARGET LINE POLYNOMIAL

The horizontal and vertical target lines have one- and two-dimensional components. In order to avoid table look-ups in the flight software, it is desired to functionalize the target line in a polynomial form. The following sections discuss the polynomials used for this purpose.

Univariate Polynomial for Functionalization in One Dimension

Lagrange Interpolating Polynomial Lagrange polynomials are numerically better behaved than power-series polynomials. For a set of N data points, the Lagrange interpolation formula is:⁵

$$y(x) = \sum_{j=1}^N y_j \phi_j(x) \quad (2)$$

where

$$\begin{aligned} y(x) &= \text{the approximating polynomial} \\ y_j &= \text{the value of } y \text{ at } x_j \\ \phi_j(x) &= \text{the set of interpolating functions} \end{aligned}$$

The set of interpolating functions are polynomials of order $N-1$, defined as:

$$\phi_j(x) = \prod_{\substack{m=1 \\ m \neq j}}^N \frac{(x - x_m)}{(x_j - x_m)} \quad (3)$$

Optimal Node Spacing When using polynomial interpolation, equispaced node points exacerbate the Runge phenomenon and can cause large errors between the node points. In order to minimize the approximation error between the nodes, approximation theory states that it is best to use unevenly spaced points, such as points defined by the roots of orthogonal polynomials. For a more detailed discussion of this topic, see Chapter 5 of reference 6. One such set of node points is the Chebyshev-Gauss-Lobatto (CGL) points:

$$x_j = \cos\left(\frac{(j-1)\pi}{N-1}\right), j = 1, 2, \dots, N \quad (4)$$

These node points are defined on the domain $[-1,1]$. It is desired to use these points to define the node spacing for the real parameters of the EI target line. This can be accomplished by mapping the CGL optimal node spacing points to the real parameter domain with the following equation:⁷

$$x_j = \frac{(x_N - x_1) X_{CGLj} + (x_N + x_1)}{2} \quad (5)$$

where

$$\begin{aligned} x_j &= \text{the } j^{th} \text{ real EI target line parameter} \\ X_{CGLj} &= \text{the } j^{th} X_{CGL} \text{ node point} \end{aligned}$$

Multivariable Polynomial for Functionalization in Two Dimensions

Consider the generic multivariable polynomial in two variables:

$$z(x, y) = \sum_{i=1}^{N_x+1} \sum_{j=1}^{N_y+1} C_{i,j} x^{(i-1)} y^{(j-1)} \quad (6)$$

where

$$\begin{aligned} z(x, y) &= \text{the approximating polynomial} \\ N_x &= \text{highest order of } x \\ N_y &= \text{highest order of } y \\ C_{i,j} &= \text{coefficient } i, j \end{aligned}$$

This equation defines a surface z as a function of the variables x and y and can be used to define a smooth function of two variables.

EI Target Polynomials

Horizontal Target Line The horizontal target line consists of the geodetic latitude, longitude, and inertial topocentric azimuth at EI. Lagrange polynomials with 60 node points are defined for the longitude and azimuth using geodetic latitude as the independent variable. Figure 6 shows the parameters of the horizontal target line. In each plot, the blue line is the Lagrange polynomial, and the red circles are the node points defining the Lagrange polynomial.

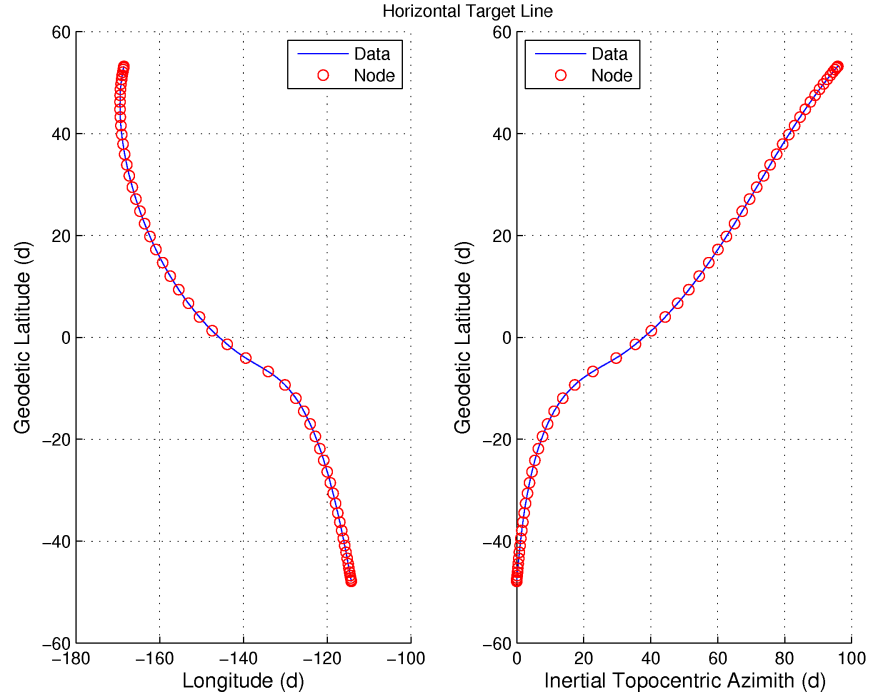


Figure 6. Horizontal Target Line

Vertical Target Line The vertical target line consists of the inertial topocentric flight path angle basis β_γ and delta Δ_γ functions. The basis function is defined as a multivariable polynomial of order 4 in both geodetic latitude and azimuth. The basis function data is fit to the surface equation using unconstrained minimization of the error norm. The delta function is defined as a Lagrange polynomial with 40 node points using inertial velocity magnitude as the independent variable. The basis function is shown in Figure 3. Figure 7 shows the basis function surface fit errors. The errors are well within an allowable tolerance. Figure 8 shows the delta function. In this plot, the blue line is the Lagrange polynomial, and the red circles are the node points defining the Lagrange polynomial.

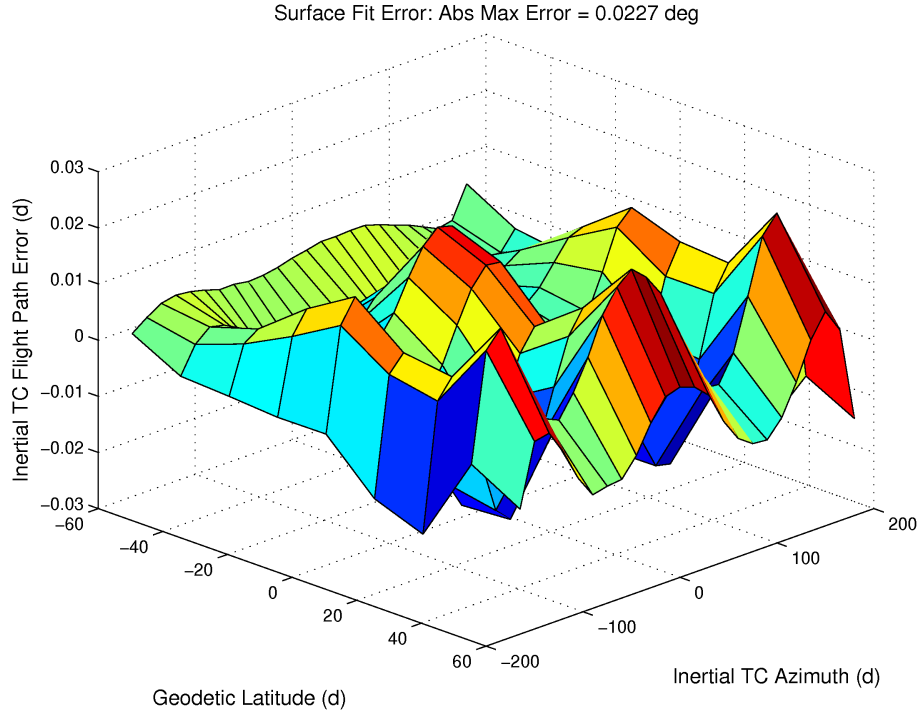


Figure 7. Vertical Target Line Basis Function Fit Error

SUMMARY

An EI target line has been developed for Orion that functionalizes a complex set of entry trajectory constraints into a set of polynomials. These polynomials will be used in both the on-board targeting algorithm and the ground-based trajectory optimization programs that define acceptable return trajectories to Earth.

REFERENCES

- [1] J. Broome and W. Johnson, "Orion Entry, Descent, and Landing Performance and Mission Design," *AIAA Guidance, Navigation and Control Conference*, August 2007. AIAA Paper 2007-6430.
- [2] J. Williams, E. C. Davis, D. E. Lee, G. L. Condon, T. F. Dawn, and M. Qu, "Global Performance Characterization of the Three Burn Trans-Earth Injection Maneuver Sequence Over the Lunar Nodal Cycle," *Proceedings of the 2009 Astrodynamics Specialist AAS/AIAA Joint Conference*, 2009. AAS Paper 09-380.

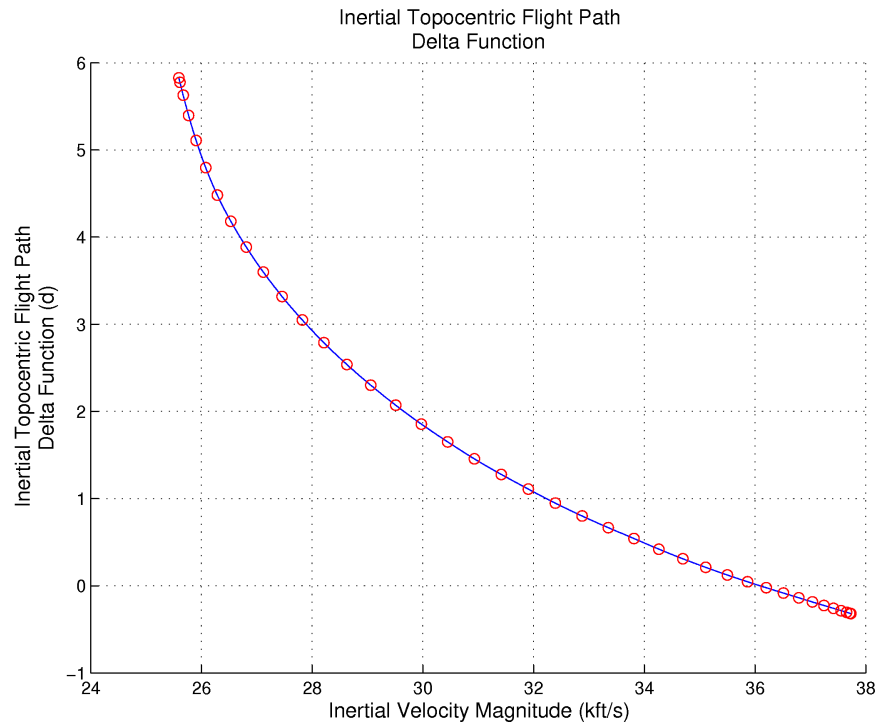


Figure 8. Vertical Target Line Delta Function

- [3] R. J. Whitley, C. A. Ocampo, and J. Williams, “Performance of an Autonomous Multi-Maneuver Algorithm for Lunar Trans-Earth Injection,” *Journal of Spacecraft and Rockets*, Vol. 49, No. 1, 2012, pp. 165–174.
- [4] M. Frostad, “Documentation of Lunar Return Entry Interface Tool v8,” Tech. Rep. NASA Tech. Rep. FltDyn-CEV-10-50, NASA Johnson Space Center, March 2010.
- [5] D. Zwillinger, ed., *CRC Standard Mathematical Tables and Formulae*. Boca Raton, Florida: CRC Press, 30th ed., 1996.
- [6] L. N. Trefethen, *Spectral Methods in Matlab*. Philadelphia, Pennsylvania: Society for Industrial and Applied Mathematics, 2000.
- [7] J. R. Rea, “A Legendre Pseudospectral Method for Rapid Optimization of Launch Vehicle Trajectories,” Master’s thesis, Massachusetts Institute of Technology, May 2001.